Determination of stress-strain curves of sheet metals by hydraulic bulge test

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Abstract. The paper is focused on the determination of the biaxial stress – strain curves by hydraulic bulging tests with circular die using a new methodology. This methodology is based on a modified version of Kruglov's formula for the calculation of the polar thickness that takes into account the non-uniformity of the strain distribution on the dome surface. In order to validate the methodology, the authors have performed both stepwise and continuous bulging experiments using the optical measurement system ARAMIS. The comparison with experimental data shows an improved accuracy of the modified Kruglov's formula.

Keywords: hydraulic bulge test, stress-strain curve, analytical model.
PACS: 62.20.F; 62.20.D; 62.20.fq

INTRODUCTION

The successful implementation of the finite element simulations in the design phase of sheet metal forming processes depends on the accuracy of the material characteristics. From this point of view, the hardening law defining the relationship between the flow stress and the plastic strain has a major influence on the quality of the numerical results. The use of hydraulic bulge test for the determination of the stress-strain curve gains more and more attention from the sheet metal forming industry. The most important advantage of the bulge test is its capability to attain very high levels of straining [1]. The main problem of using the bulge test for the determination the stress-strain relationship is the measurement of the bulge radius and polar thickness of the specimen. This data is required to calculate the stress based on the membrane theory. Extensive efforts have been made to develop analytical models for the calculation of the dome radius and thickness. Hill [2] developed an analytical model of the hydraulic bulging process. He admitted the spherical shape of the dome and neglected the influence of the fillet radii located at the entrance of the insert die. Panknin [3] also proposed a formula for the calculation of the curvature radius. This relationship takes into account the effect of the fillet radius on the dimensional characteristics of the dome. By comparing the analytical results with experimental data, Panknin noticed deviations less than 10% of the calculated curvature radius as compared to the experimental value. Chakrabarty and Alexander [4] improved the accuracy of the formulas previously proposed by Hill by taking into account the hardening effects. Gologranc [5] noticed that the values of the polar thickness predicted by Hill's formula were considerably different from his own experimental results. Shang [6] extended the analytical models developed by Hill in order to take into account the fillet radius of the die insert. According to his experimental observations, the value of the fillet radius has a small influence on the polar strains. Atkinson [7] also tried to improve the accuracy of the analytical predictions referring to the polar thickness and dome radius. Kruglov [8] developed a formula for the calculation of the polar strains. This formula is based on the assumption that the meridian strain is uniformly distributed on the dome surface. Analytical models for the computation of the pressure-time relationship were developed by Banabic [9] for the bulging of both strain hardening and superplastic materials trough elliptical dies and by Vulcan [10] and Banabic [11] for superplastic forming of aluminium sheets using the cone-cup testing method.

This paper presents a recently proposed analytical approach for the accurate and efficient determination of the biaxial stress – strain curves by hydraulic bulge test with circular dies [12]. The use of this methodology is illustrated in the case of a DC04 steel sheet.

METHODOLOGY FOR THE DETERMINATION OF STRESS-STRAIN CURVES

The methodology used to determine the stress-strain curve is based on a modified version of Kruglov's formula for calculation of the polar thickness. The modification consists in taking into account the non-uniform distribution of the strains on the specimen surface by means of a correction coefficient. This approach is briefly described below but more details can be found in [12].

Analytical modelling

Figure 1 shows the geometric configuration of the deformed specimen used for the analytical modelling of the hydraulic bulging test. Table 1 summarizes the notations in Fig. 1.

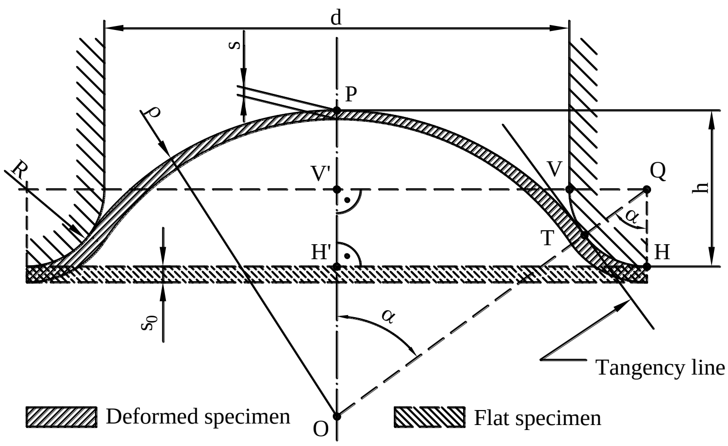

FIGURE 1. Schematic representation of the specimen subjected to hydraulic bulging

The current value of the biaxial surface stress (σ_b) is defined by Laplace's formula:

$$\sigma_b = \frac{p\rho}{2s}, \tag{1}$$

where (p) is the internal pressure.

The corresponding thickness strain (the so-called biaxial strain ε_b) can be evaluated as follows:

$$\varepsilon_b = \ln\frac{s_0}{s}. \tag{2}$$

Equations (1) and (2) can be used to obtain a biaxial stress - strain diagram only if the quantities p, ρ and s are either measured or derived from other experimental data. The pressure p can be easily measured using a sensor connected to the hydraulic chamber of the experimental device. The other process variables, namely the curvature radius ρ and the polar thickness s are less accessible to the direct determination. It is more convenient to obtain their values in an indirect manner, using approximate formulas that involve the current value of the polar height h.

TABLE 1. Geometric parameters for analytical modelling of hydraulic bulging.

Parameter	Desription
D	Diameter of the bulging orifice
R	Fillet radius of the bulging orifice
S_0	Initial (nominal) thickness of the specimen
S	Current thickness of the specimen in the polar region (point P)
ρ	Current radius of the dome surface
H	Height defining the current position of the pole P
α	Angle spanned by the arcs TP and TH respectively (α is expressed in radians)

The curvature radius ρ can be evaluated with Panknin's formula [3]

$$\rho = \frac{1}{2h}\left(\frac{d}{2}+R\right)^2 + \frac{h}{2} - R\,. \tag{3}$$

The experimental studies performed by other researchers [13] proved that amongst the numerous relationships that can be used to compute the current value of the polar thickness *s* Kruglov's formula [8] provides the best results. This relationship reads

$$s = s_0 \exp\left(-\varepsilon_b\right) = s_0 \left(\frac{\alpha}{\sin\alpha}\right)^{-2}, \tag{4}$$

where (α) can be calculate using the dome height h and the dimensional characteristics of the experimental device (i.e. diameter and the fillet radius of the bulging orifice)

$$\alpha = \arcsin\left[\left(\frac{d}{2}+R\right)\Bigg/\left(\frac{1}{2h}\left(\frac{d}{2}+R\right)^2 + \frac{h}{2}\right)\right]. \tag{5}$$

The accuracy of Kruglov's formula is still improvable if Eq (4) is modified as follows:

$$s = s_0 \exp\left(-\varepsilon_b\right) = s_0 \left(\frac{\alpha}{\sin\alpha}\right)^{-2(1+c\alpha)}. \tag{6}$$

The coefficient c is a strictly positive constant that takes into account the non-uniformity of the meridian strain distribution on the dome surface. More details about the development of the analytical expression of the coefficient c can be found in [12]. The calculation of the parameter c can be done using the following formula

$$c = \left(\ln\sqrt{\frac{s_0}{s_{\min}}} - \ln\frac{\alpha_{\max}}{\sin\alpha_{\max}}\right)\Bigg/\left(\alpha_{\max}\ln\frac{\alpha_{\max}}{\sin\alpha_{\max}}\right), \tag{7}$$

where $s_{\min}$ is the final value of the polar thickness and $\alpha_{\max}$ is the angle spanned by the dome surface. Both quantities correspond to the final stage of the bulging process. The angle $\alpha_{\max}$ can be calculated using the equation

$$\alpha_{\max} = \arcsin\left[\left(\frac{d}{2}+R\right)\Bigg/\left(\frac{1}{2h_{\max}}\left(\frac{d}{2}+R\right)^2 + \frac{h_{\max}}{2}\right)\right], \tag{8}$$

where $h_{\max}$ is the maximum polar height measured at the end of the bulge test.

Methodology

An overview of the methodology for determination of stress-strain curve is described by the flow chart shown in Fig. 2. This methodology consists in the following steps:

Step 1: During the hydraulic bulging experiments the internal pressure and dome height are recorded continuously until the end of the test.

Step 2: The dome height and the dimensional characteristics of the experimental device (i.e. diameter of the die aperture, and the fillet radius of the die) will be used to calculate the dome radius (Eq (3)).

Step 3: The maximum dome height $h_{\max}$, the minimum polar thickness $s_{\min}$, the initial thickness of the specimen and the dimensional characteristics of the experimental device will be used to calculate the correction coefficient c (Eqs (8) and (7)).

Step 4: The dome height, the correction coefficient, the initial thickness of the specimen and the dimensional characteristics of the experimental device will be used to calculate the polar thickness *s* (Eqs (5) and (6)).

Step 5: The biaxial stress and strain will be calculated on the basis of internal pressure, polar radius, polar thickness and initial thickness of the specimen (Eqs (1) and (2)).

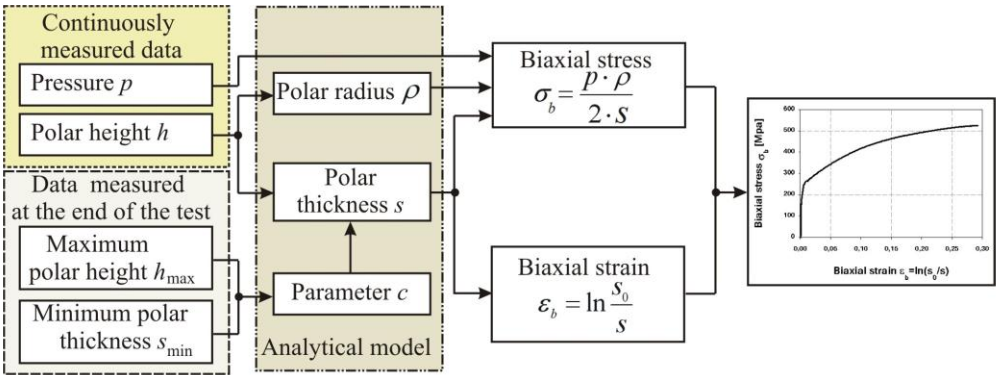

FIGURE 2. Methodology used for the determination of the biaxial stress-strain curve

EXPERIMENTAL PROCEDURE

The experiments have been performed using a hydraulic bulging device and a 3D optical measurement system ARAMIS. The geometric dimensions of the tool are (see Fig. 1): diameter of the die aperture d = 80 mm; fillet radius of the die R = 7 mm. A DC04 steel sheet with an initial thickness of 0.85 mm (EN 10130) was chosen for the experiments.

Two types of experiments have been performed for the validation of the new methodology. The first test consists in stepwise bulging experiments. In the case of these tests, the process has been stopped at different values of the polar height. The corresponding level of the bulging pressure has been also recorded. After removing the specimen from the bulging device, the deformed surface has been inspected with a 3D Coordinate Measuring Machine, with the aim of determining its minimum thickness, polar height and curvature radius. The maximum deviation from the spherical shape, as established by 3D CMM, is 0.114 mm, while the minimum deviation is 0.062 mm. These experimental results have been used for the calculation of discrete points belonging to the biaxial stress – strain curve.

The second type of tests consists in continuous bulging experiments using a 3D optical measurement system ARAMIS. In this case, the pressure and polar height has been recorded by a pressure sensor and two CCD cameras, respectively. The experimental data has been used for the determination of biaxial stress – strain curves.

RESULTS AND DISCUSSION

Bulge radius and dome height

Figure 3.a shows the variation of the bulge radius given by Eq (3) as a function of the dome height recorded continuously with the ARAMIS system and Fig. 3.b shows the variation of the pressure as a function of the dome height. The diagrams from Figs. 3.a and 3.b also contains the discrete values provided by the 3D CMM in the case of the stepwise experiments. As one may notice from Figs. 3.a and 3.b, there are no significant differences between the continuous and stepwise measurements. The maximum value of the pressure attained during the stepwise bulging tests was 11.87 MPa. The polar height corresponding to this load was 21.81 mm.

Biaxial strain

Figures 4.a and 4.b show the variation of the biaxial strain as a function of the dome height and as a function of the bulge radius, respectively. The data presented on the diagrams was obtained both by measurement and calculations. As noticeable from Fig. 4.a, up to a polar height of about 5 mm, both curves obtained from the standard and modified Kruglov's formulas are almost coincident with the data provided by the ARAMIS system.

For larger values of the polar height, the variation predicted by the standard Kruglov's formula gradually deviates from the experimental curve, while the predictions of the modified formula remain in a closer neighbourhood. Figure 4.b shows an inverse trend: at the high levels of the bulge radius both curves obtained from the standard and modified Kruglov's formulas are close to the experimental curve obtained with the ARAMIS system. As the bulge radius decreases, the results obtained by standard Kruglov's formula gradually deviates from the experimental curve, while the results obtained with the modified Kruglov's formula shows a good agreement with the experimental data.

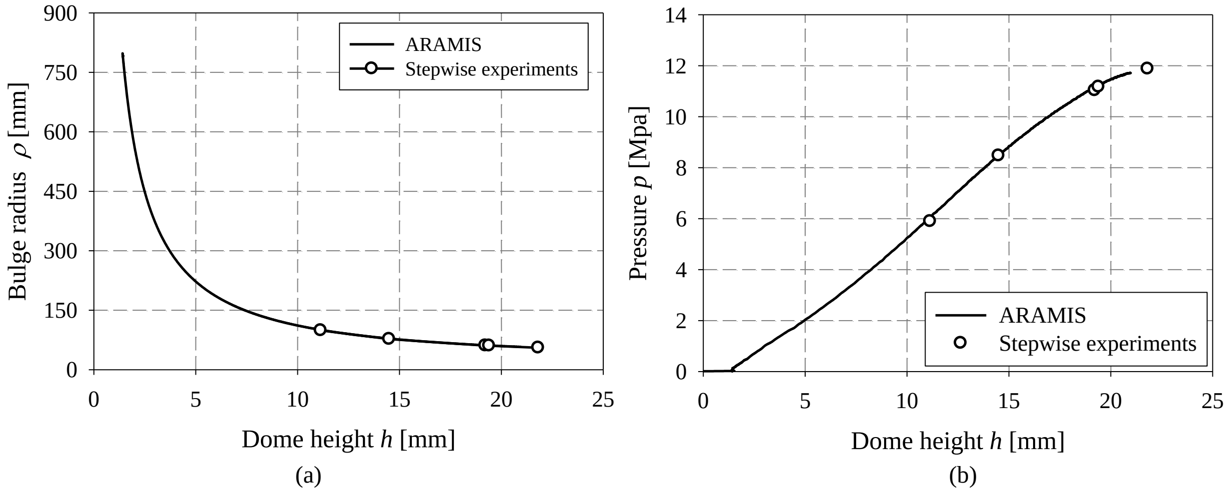

FIGURE 3. Bulge radius vs dome height (a); Hydraulic pressure vs dome height (b)

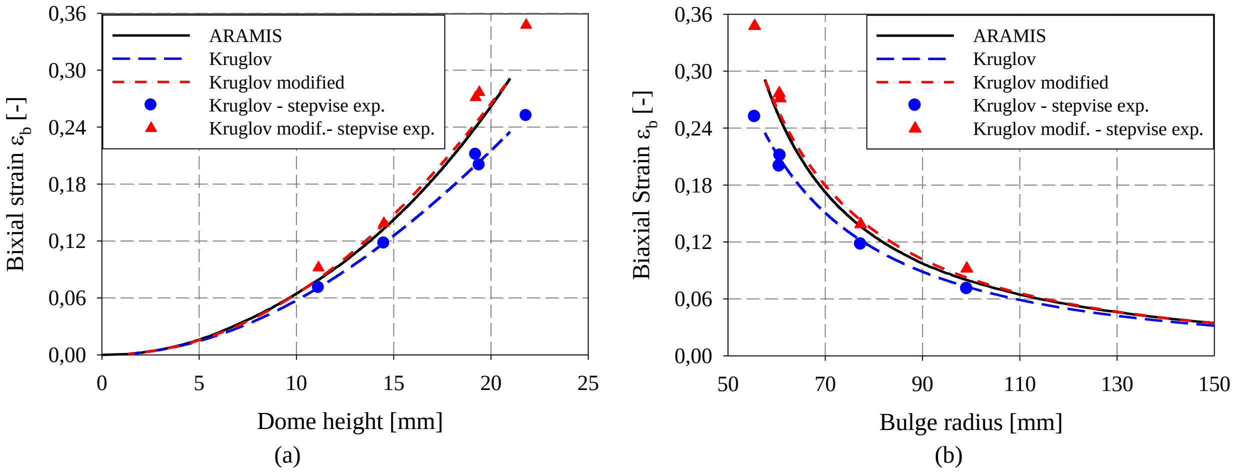

FIGURE 4. Biaxial strain vs dome height (a); Biaxial strain vs bulge radius (b)

Biaxial stress – strain curves

A similar comparison has been performed in the case of the biaxial stress – strain curves. As noticeable from Fig. 5, the results obtained using the Kruglov's modified formula are in better agreement with the experimental data provided by the ARAMIS system. The accuracy of the predictions remains very good up to the end of the experimental curves. In contrast, the standard Kruglov's formula tends to underestimate both the biaxial strains and stresses in the final stages of the bulging experiment (see the detail in Fig. 5). The correction included in the modified relationship is able to drag the calculated curve closer to the experimental data provided by the ARAMIS measurement system. The same conclusion can be drawn when comparing the predictions of the modified Kruglov's formula with the results obtained from stepwise experiments.

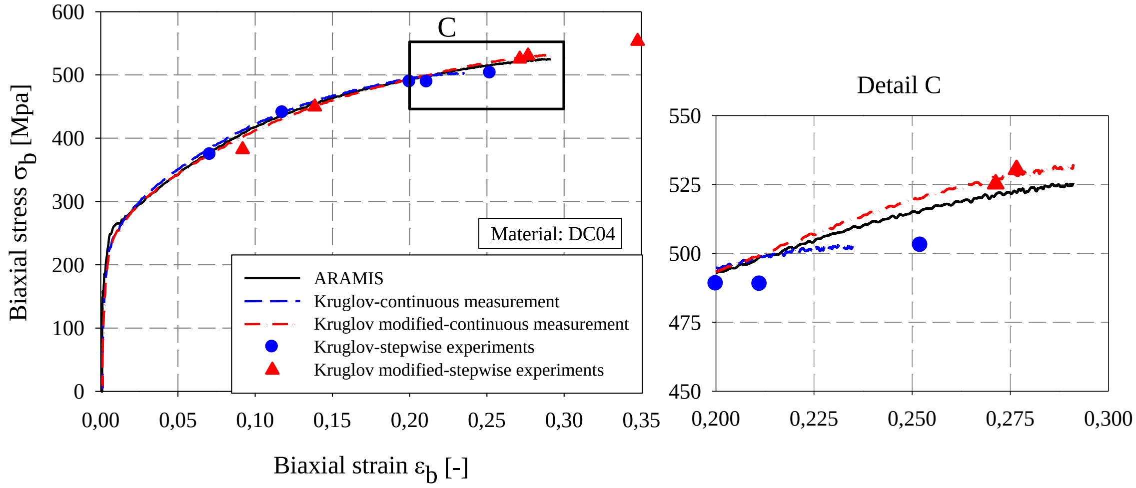

FIGURE 5. Biaxial stress-strain curves

CONCLUSION

A new methodology has been developed for the experimental determination of the biaxial stress – strain curves. The methodology is based on a modified version of Kruglov’s formula for the calculation of the polar thickness that takes into account the non-uniformity of the strain distribution on the dome surface by means of a correction coefficient. It was shown that the results provided by the modified Kruglov's formula are in very good agreement with the experimental data obtained with the ARAMIS system. Due to its accuracy and simplicity, the methodology can be easily implemented in the industrial laboratories involved in the testing of sheet metals.

ACKNOWLEDGMENTS

This paper was supported by the European Commission, within the project: “Develop and support multidisciplinary postdoctoral programs in primordial technical areas of national strategy of the research - development - innovation" 4D-POSTDOC, contract nr. POSDRU/89/1.5/S/52603, project co-funded from European Social Fund through Sectorial Operational Program Human Resources 2007-2013 and by the Romanian National University Research Council (CNCSIS), within the program IDEI, PCCE, Grant No. 100/2010.

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